AOR Questions Unit 1 & Unit 2 Theory questions

Q1. Discuss the step involved in the process of decision making? (may 2017)

Ans. The Steps involved are as follows:

1. Identification of the problem in real sense by the decision maker
2. Alternative courses of action or strategies
3. Identify the expected future events(not under the control of decision maker)
4. Constructing the payoff table
5. Selecting optimum decision criteria

Q2. Difference between PERT and CPM. Under what circumstances would you consider PERT as opposed to CPM in project management? (May 2017)

Ans.

CPM	PERT
1. It is deterministic model under which the result is ascertained in a manner of certainty	It is probabilistic model under which the result is estimated in the manner of probability
2. It deals with the activities of precise well know time.	It deals with the activities of uncertain time.
3. It is used for repetitive jobs like residential construction.	It is known for non – repetitive jobs like planning and scheduling of research programmes.
4. It is activity oriented in as much as its results are calculated on the basis of activities	It is event oriented in as much as its results are calculated on the basis of events.
5. It does not make use of dummy activities	It make use of dummy activities to represent the proper sequencing of the activities.
6. It deals with cost of project schedules and their minimizations	It has nothing to do with cost of the project
7. Deals with the concept of crashing	Does not deals with concept of crashing
8. It does not make use of statistical devices.	It make use of statistical devices.

We consider PERT as opposed to CPM in project management:

1. PERT was developed in connection with an R&D work. Therefore, it had to cope with the uncertainties that are associated with R&D activities. In PERT, the total project duration is regarded as a random variable. Therefore, associated probabilities are calculated so as to characterise it.
2. It is an event-oriented network because in the analysis of a network, emphasis is given on the important stages of completion of a task rather than the activities required to be performed to reach a particular event or task.
3. PERT is normally used for projects involving activities of non-repetitive nature in which time estimates are uncertain.
4. It helps in pinpointing critical areas in a project so that necessary adjustment can be made to meet the scheduled completion date of the project.

Q3) What do you understand by Decision Tree Analysis? How is decision tree drawn and how is such analysis useful in decision making? Explain taking examples (Dec 2015)

Ans:
A decision tree analysis is a map of the possible outcomes of a series of related choices. It allows an individual or organization to weigh possible actions against one another based on their costs, probabilities, and benefits.

How is a decision tree drawn

A decision tree typically starts with a single node, which branches into possible outcomes. Each of those outcomes leads to additional nodes, which branch off into other possibilities. This gives it a treelike shape.

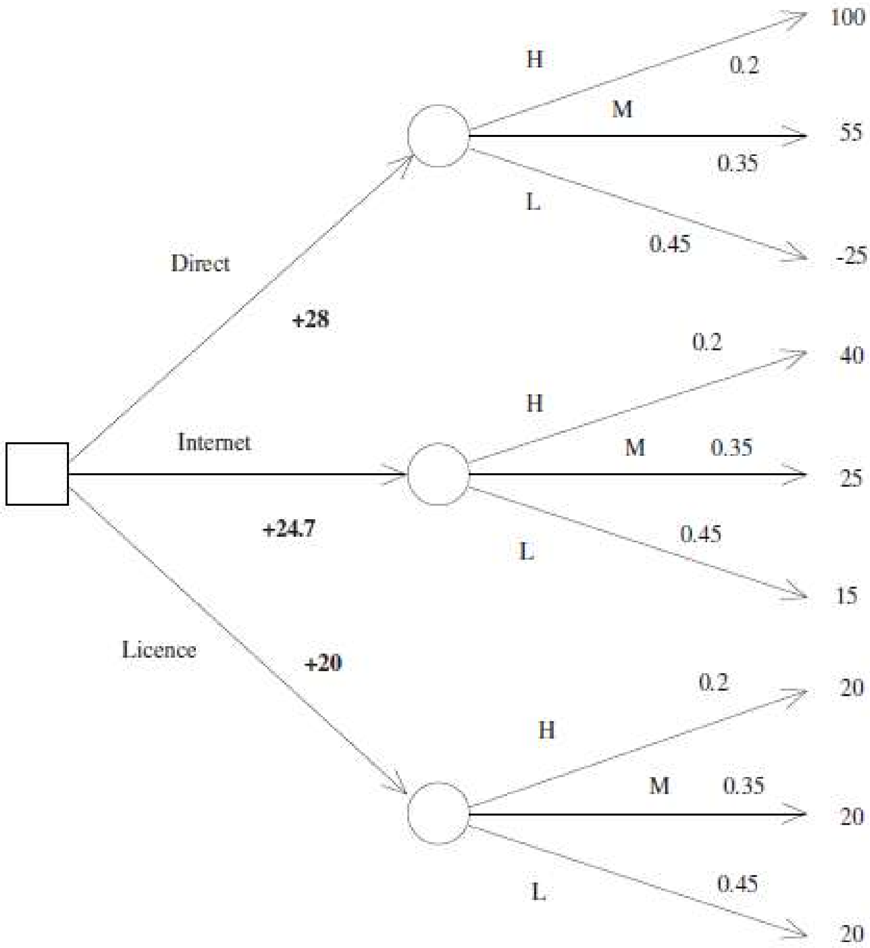
There are three different types of nodes: chance nodes, decision nodes, and end nodes. A chance node, represented by a circle, shows the probabilities of certain results. A decision node, represented by a square, shows a decision to be made, and an end node shows the final outcome of a decision path.

Decision tree analysis example

Synaptec is a small technology company with a new and innovative product that they wish to launch on to the market. It could go for a direct approach, launching onto the whole of the domestic market through traditional distribution channels, or it could launch only on the internet. A third option exists where the product is licensed to a larger company through the payment of a licence fee irrespective of the success of the product. How should the company launch the product? The company has done some initial market research and the managing director, Jack Holmes, believes the demand for the product can be classed into three categories: high, medium or low. Jack thinks that these categories will occur with probabilities 0.2, 0.35and 0.45 respectively and his thoughts on the likely profits (in £K) to be earned in each plan are High Medium Low

Direct 100 55 -25
Internet 46 25 15
Licence 20 20 20

The EMV of each plan can be calculated as follows:
EMV (Direct) = $0.2 \times 100 + 0.35 \times 55 + 0.45 \times (-25)$ = £28K
EMV (Internet) = $0.2 \times 46 + 0.35 \times 25 + 0.45 \times 15$ = £24.7K
EMV (Licence) = $0.2 \times 20 + 0.35 \times 20 + 0.45 \times 20$ = £20K.
On the basis of expected monetary value, the best choice is the Direct approach as this maximises his EMV.



Q4. Explain briefly various Applications of AOR? What are advantages and disadvantages of AOR studies? (DEC 2014)

Ans**. Applications of AOR**

O.R. is a problem solving and decision taking technique. It is considered a kit of scientific and programmable rules which provides the management a "quantitative basis" for decisions concerning the operation under its control.

1. Allocation and Distribution in Projects:

(i) Optimal allocation of resources such as men materials machines, time and money to projects.

(ii) Determination and deployment of proper workforce.

(iii) Project scheduling, monitoring and control.

2. Production and Facilities Planning:

(i) Factory size and location decision.

(ii) Estimation of number of facilities required.

(iii) Preparation of forecasts for the various inventory items and computation of economic order quantities and reorder levels.

(iv) Scheduling and sequencing of production runs by proper allocation of machines.

(v) Transportation loading and unloading,

(vi) Warehouse location decision.

(vii) Maintenance policy decisions.

3. Programmes Decisions:

(i) What, when and how to purchase to minimize procurement cost.

(ii) Bidding and replacement policies.

4. Marketing:

(i) Advertising budget allocation.

(ii) Product introduction timing.

(iii) Selection of advertising media.

(iv) Selection of product mix.

(v) Customer’s preference of size, colour and packaging of various products.

5. Organization Behaviour:

(i) Selection of personnel, determination of retirement age and skills.

(ii) Recruitment policies and assignment of jobs.

(iii) Recruitment of employees.

(iv) Scheduling of training programs.

6. Finance:

(i) Capital requirements, cash flow analysis.

(ii) Credit policies, credit risks etc.

(iii) Investment decision.

(iv) Profit plan for the company.

7. Research and Development:

(i) Product introduction planning.

(ii) Control of R&D projects.

(iii) Determination of areas for research and development.

(iv) Selection of projects and preparation of their budgets.

(v) Reliability and control of development projects thus it may be concluded that operation research can be widely utilized in management decisions and can also be used as corrective measure.

Advantages of AOR

1. **Better Systems:** Often, an O.R. approach is initiated to analyze a particular problem of decision making such as best location for factories, whether to open a new warehouse, etc. It also helps in selecting economical means of transportation, jobs sequencing, production scheduling, replacement of old machinery, etc.
2. **Better Control:** The management of large organizations recognize that it is a difficult and costly affair to provide continuous executive supervision to every routine work. An O.R. approach may provide the executive with an analytical and quantitative basis to identify the problem area. The most frequently adopted applications in this category deal with production scheduling and inventory replenishment.
3. **Better Decisions:** O.R. models help in improved decision making and reduce the risk of making erroneous decisions. O.R. approach gives the executive an improved insight into how he makes his decisions.

Disadvantages of AOR

1. **Dependence on an Electronic Computer:** O.R. techniques try to find out an optimal solution taking into account all the factors. In the modern society, these factors are enormous and expressing them in quantity and establishing relationships among these require voluminous calculations that can only be handled by computers.

2. **Non-Quantifiable Factors:**O.R. techniques provide a solution only when all the elements related to a problem can be quantified. All relevant variables do not lend

themselves to quantification. Factors that cannot be quantified find no place in O.R. models.
3. **Distance between Manager and Operations Researcher:** O.R. being specialist's job requires a mathematician or a statistician, who might not be aware of the business problems. Similarly, a manager fails to understand the complex working of O.R. Thus, there is a gap between the two.
4. **Money and Time Costs:** When the basic data are subjected to frequent changes, incorporating them into the O.R. models is a costly affair. Moreover, a fairly good solution at present may be more desirable than a perfect O.R. solution available after sometime.
5. **Implementation:** Implementation of decisions is a delicate task. It must take into account the complexities of human relations and behaviour.

Q 5. **Explain Linear Programming model for product-mix with the help of a suitable example. (Dec 2016)**

Ans. Linear programming (LP) can be defined as a mathematical technique for determining the best allocation of a firm's limited resources to achieve optimum goal. It is also a mathematical technique used in Operation Research (OR) or Management Sciences to solve specific types of problems such as allocation, transportation and assignment problems that permits a choice or choices between alternative courses of action.

Product Mix Example

This part of our Solver Tutorial takes you **step by step** through the process of creating a Solver model, using a Product Mix example. We'll first show you how to define the problem and write out formulas for the objective and constraints. Then we'll walk you through two ways to define and solve your model -- in an **Excel spreadsheet**, or in a **Visual Basic program**.

The Example Problem

Imagine that you manage a factory that produces four different types of wood paneling. Each type of paneling is made by gluing and pressing together a different mixture of pine and oak chips. The following table summarizes the required amount of gluing, pressing, and mixture of wood chips required to produce a pallet of 50 units of each type of paneling:

	Resources Required per Pallet of Paneling Type			
	Tahoe	**Pacific**	**Savannah**	**Aspen**
Glue (quarts)	50	50	100	50
Pressing (hours)	5	15	10	5
Pine chips (pounds)	500	400	300	200
Oak chips (pounds)	500	750	250	500

In the next production cycle, you have 5,800 quarts of glue; 730 hours of pressing capacity; 29,200 pounds of pine chips; and 60,500 pounds of oak chips available. Further assume that each pallet of Tahoe, Pacific, Savannah, and Aspen panels can be sold for profits of $450, $1,150, $800, and $400, respectively.

Before we implement this problem statement in either Excel or Visual Basic, let's **write out formulas** corresponding to the verbal description above. If we temporarily use the symbol X_1 for the number of Tahoe pallets produced, X_2 for the number of Pacific pallets produced, and X_3 for the number of Savannah pallets produced, and X_4 for the number of Aspen pallets produced, the objective (calculating total profit) is:

Maximize: $450 X_1 + 1150 X_2 + 800 X_3 + 400 X_4$

A pallet of each type of panel requires a certain amount of glue, pressing, pine chips, and oak chips. The amount of resources used (calculated by the left hand side of each constraint) depends on the mix of products built, and we have a limited amount of each type of resource available (corresponding to the constraint right hand side values). The constraints for this problem are expressed as follows:

Subject to:

$50 X_1 + 50 X_2 + 100 X_3 + 50 X_4 <= 5800$ (Glue)
$5 X_1 + 15 X_2 + 10 X_3 + 5 X_4 <= 730$ (Pressing)
$500 X_1 + 400 X_2 + 300 X_3 + 200 X_4 <= 29200$ (Pine chips)
$500 X_1 + 750 X_2 + 250 X_3 + 500 X_4 <= 60500$ (Oak chips)

Since the number of products built cannot be negative, we'll also have **non-negativity conditions** on the variables:

$X_1, X_2, X_3, X_4 >= 0.$

Q 6. Define a) Slack, surplus, artificial variables b) equality and inequality c) Decision variables and basic variables? (Dec 2015)

Ans**. A) SlackVariables**

Slack variable represents an unused quaintly of resources,it is added to less than or equal ($\leq$) to type constraints in order to get an equality constraint.

Surplus Variables

A surplus variable represents the amount by which solution values exceed a resource. These variables are also called 'Negative Slack Variables' . Surplus variables like slack variables carry a zero coefficient in the objective function. it is added to greater than or equal to ($\geq$) type constraints in order to get an equality constraint.

Artificial Variables :

Artificial variables are added to those constraints with equality (=) and greater than or equal to ($\geq$) sign. An Artificial variable is added to the constraints to get an initial solution to an LP problem. Artificial variables have no meaning in a physical sense and are not only used as a tool for generating an initial solution to an LP problem.

b) **Equality and inequality**

Slack variable is a variable that is added to an inequality constraint to transform it into an equality. Introducing a slack variable replaces an inequality constraint with an equality constraint and a non negativity constraint.

Example:

By introducing the slack variable y>= 0, the inequalityAx<=bcan be converted to the equation Ax+y = b.

c) **Decision Variables**

A decision variable is a quantity that the decision-maker controls. For example, the number of nurses to employ during the morning shift in an emergency room may be a decision variable in an optimization model for labor scheduling. The OptQuest Engine manipulates decision variables in search of values that produce the optimal value for the objective function.

Basic Variables

Each solution to any system of equations is called a Basic Solution (BS). Those BS, which are feasible are called Basic Feasible Solutions (BFS).
In every basic solution, the variables, which you set equal to zero are called the Non-Basic Variables (NBV), all other variables you compute by using the system of equations are called Basic Variables (BV).

Q7.) Explain in detail, any one method for solving a transportation problem. Would you recommend this method to solve an assignment problem? (Dec 2014)

Ans. There is a type of linear programming problem that may be solved using a simplified version of the simplex technique called **transportation method**. Because of its major application in solving problems involving several product sources and several destinations of products, this type of problem is frequently called the **transportation problem**.

We will now discuss each one in the context of a simple example. Suppose one company has four factories supplying four warehouses and its management wants to determine the minimum-cost shipping schedule for its weekly output of chests. Factory supply, warehouse demands, and shipping costs per one chest (unit) are shown in Table 7.1

				Shipping Cost per Unit (in $)				
Factory	Supply	Warehouse	Demand	From	To E	To F	To G	To H
A	15	E	10	A	10	30	25	15
B	6	F	12	B	20	15	20	10
C	14	G	15	C	10	30	20	20
D	11	H	9	D	30	40	35	45

The Table below shows a **northwest-corner assignment**. (Cell A-E was assigned first, A-F second, B-F third, and so forth.) Total cost : 10*10 + 30*4 + 15*10 + 30*1 + 20*12 + 20*2 + 45*12 + 0*1 = 1220($).

From \ To	E	F	G	H	Factory Supply
A	10 **10**	30 **4**	25	15	14
B	20	15 **10**	20	10	10
C	10	30 **1**	20 **12**	20 **2**	15
D	30	40	35	45 **12**	12
Dummy	0	0	0	0 **1**	1
Destination Requirements	10	15	12	15	52 52

No we would not recommend to solve this problem using assignment problem as it would be very tedious and consume lot of time.

Q8 **Write a short note on travelling sales man problem? (May 2016)**

Ans. The **travelling salesman problem** (**TSP**) asks the following question: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?" It is an NP-hardproblem in combinatorial optimization, important in operations research

The travelling purchaser problem and the vehicle routing problem are both generalizations of TSP.

The Travelling Salesman Problem describes a salesman who must travel between N cities. The order in which he does so is something he does not care about, as long as he visits each once during his trip, and finishes where he was at first. Each city is connected to other close by cities, or nodes, by airplanes, or by road or railway.Each of those links between the cities has one or more weights (or the cost) attached. The cost describes how "difficult" it is to traverse this edge on the graph, and may be given, for example, by the cost of an airplane ticket or train ticket, or perhaps by the length of the edge, or time required to complete the traversal. The salesman wants to keep both the travel costs, as well as the distance he travels as low as possible.

The Traveling Salesman Problem is typical of a large class of "hard" optimization problems that have intrigued mathematicians and computer scientists for years. Most important, it has applications in science and engineering.

For example, in the manufacture of a circuit board, it is important to determine the best order in which a laser will drill thousands of holes. An efficient solution to this problem reduces production costs for the manufacturer.

Q1. There are 2 players A and B and their pay-off matrix concerning zero sum two person game is given as below.

		Player B				
		I	**II**	**III**	**IV**	**V**
	I	-2	0	0	5	3
Player A	**II**	4	2	1	3	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

What is the optimal plan for both the players?

Solution.

We use the maximin (minimax) principle to analyze the game.

		Player B					
		I	**II**	**III**	**IV**	**V**	**Minimum**
	I	-2	0	0	5	3	-2
Player A	**II**	4	2	1	3	2	1
	III	-4	-3	0	-2	6	-4
	IV	5	3	-4	2	-6	-6
Maximum		5	3	1	5	6	

Select minimum from the maximum of columns.
Minimax=1

Player A will choose II strategy, which yields the maximum payoff of 1.

Select maximum from the minimum of rows.
Maximin=1
Similarly, player B will choose III strategy.

Since the value of maximin coincides with the value of the minimax, therefore, saddle point (equilibrium point) = 1.

The optimal strategies for both players are: Player A must select II strategy and player B must select III strategy. The value of game is 1, which indicates that player A will gain 1 unit and player B will sacrifice 1 unit.

Q2. Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix:

		Company B		
		I	**II**	**III**
Company A	**I**	-2	14	-2
	II	-5	-6	-4
	III	-6	20	-8

Determine the optimal strategies for both the companies.

Solution. First, we apply the maximin (minimax) principle to analyze the game.

		Company B			
		I	**II**	**III**	**Minimum**
Company A	**I**	-2	14	-2	-2
	II	-5	-6	-4	-6
	III	-6	20	-8	-8
Maximum		-2	20	-2	

Minimax=-2
Maximin = -2

There are two elements whose value is –2. Hence, the solution to such a game is not unique.

In the above problem, there is no saddle point. In such cases, the maximin and minimax principle of solving a game problem can't be applied. Under this situation, both the companies may resort to what is known as mixed strategy.

Q3. Use the principle of dominance to solve this problem.

		Player B			
		I	II	III	IV
	I	3	5	4	2
Player A	II	5	6	2	4
	III	2	1	4	0
	IV	3	3	5	2

Solution:

		Player B				
		I	**II**	**III**	**IV**	**Minimum**
	I	3	5	4	2	2
Player A	**II**	5	6	2	4	2
	III	2	1	4	0	0
	IV	3	3	5	2	2
Maximum		5	6	5	4	

There is no **saddle point** in this game.

Using Dominance Property In Game Theory

If a column is greater than another column (compare corresponding elements), then delete that column.Here, I and II column are greater than the IV column. So, player B has no incentive in using his I and II course of action.

		Player B	
		III	**IV**
	I	4	2
Player A	**II**	2	4
	III	4	0
	IV	5	2

If a row is smaller than another row (compare corresponding elements), then delete that row. Here, I and III row are smaller than IV row. So, player A has no incentive in using his I and III course of action.

		Player B	
		III	**IV**
Player A	**II**	2	4
	IV	5	2

Q4. Two jobs are to be performed on five machines A, B, C, D, and E. Processing times are given in the following table.

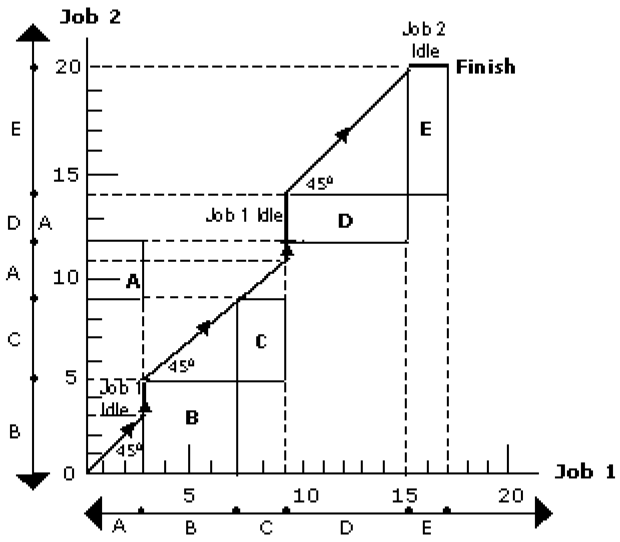
			Machine				
Job 1	**Sequence**	**:**	A	B	C	D	E
	Time	**:**	3	4	2	6	2
Job 2	**Sequence**	**:**	B	C	A	D	E
	Time	**:**	5	4	3	2	6

Use graphical method to obtain the total minimum elapsed time.

Solution:

Step 1: Mark the processing times of job 1 & job 2 on X-axis & Y-axis respectively.

Step 2: Draw the rectangular blocks by pairing the same machines as shown in the following figure.



Step 3: Starting from origin O, move through the 45^0 line until a point marked finish is obtained.

Step 4: The elapsed time can be calculated by adding the idle time for either job to the processing time for that job. In this illustration, idle time for job 1 is 5 (3+2) hours.
Elapsed time = Processing time of job 1 + Idle time of job 1
= (3 + 4 + 2 + 6 + 2) + 5 = 17 + 5 = 22 hours.
Likewise, idle time for job 2 is 2 hours.
Elapsed time = Processing time of job 2 + Idle time of job 2
= (5 + 4 + 3 + 2 + 6) + (2) = 20 + 2 = 22 hours.

Q5. The initial cost of a machine is Rs. 7100 and scrap value is Rs. 100. The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance	200	350	500	700	1000	1300	1700	2100

When should the machine be **replaced**?

Solution:

Year	Running cost	Cumulative running cost	Scrap value	Difference between initial cost and scrap value	Average investment cost / year	Average running cost / year	Average annual total cos
A	**B**	**C**	**D**	**E**	**F = E/A**	**G = C/A**	**H = F +**
1	200	200	100	7000	7000	200	7200
2	350	200 + 350 = 550	100	7000	3500	225	3775
3	500	550 + 500 = 1050	100	7000	2333.33	350	2683.33
4	700	1050 + 700 = 1750	100	7000	1750	437.5	2187.50
5	1000	1750 + 1000 = 2750	100	7000	1400	550	1950
6	1300	2750 + 1300 = 4050	100	7000	1166.67	675	1841.67
7	1700	4050 + 1700 = 5750	100	7000	1000	821.42	1821.42
8	2100	5750 + 2100 = 7850	100	7000	875	981.25	1856.25

This table shows that the average annual total cost during the seventh year is minimum. Hence, the machine should be **replaced** after the **7th** year.

Q6. The initial cost of a machine is Rs. 6100 and resale value drops as the time passes. Cost data are given in the following table:

Year	1	2	3	4	5	6	7	8
Maintenance	100	250	400	600	900	1200	1600	2000
Resale Value	800	700	600	500	400	300	200	100

When should the machine be replaced?

Solution:

Year	Running cost	Cumulative running cost	Resale value	Difference between initial	Average investment cost	Average running cost	Average annual tot

				cost and resale value	**/ year**	**/ year**	**cost**
1	100	100	800	5300	5300	100	5400
2	250	350	700	5400	2700	175	2875
3	400	750	600	5500	1833.33	250	2083.33
4	600	1350	500	5600	1400	337.5	1737.50
5	900	2250	400	5700	1140	450	1590
6	1200	3450	300	5800	966.67	575	1541.67
7	1600	5050	200	5900	842.85	721.42	1564.27
8	2000	7050	100	6000	750	881.25	1631.25

This table shows that the average annual total cost during the sixth year is minimum. Hence, the machine should be replaced after the 6^{th} year.

Q7. Describe various components of queuing system.

Answer: The various components of queuing system are:

1. **Input Source:** The input source generates customers for the service mechanism. The most important characteristic of the input source is its size. It may be either finite or infinite. Please note that the calculations are far easier for the infinite case, therefore, this assumption is often made even when the actual size is relatively large. If the population size is finite, then the analysis of queuing model becomes more involved. The statistical pattern by which calling units are generated over time must also be specified. It may be Poisson or Exponential probability distribution.

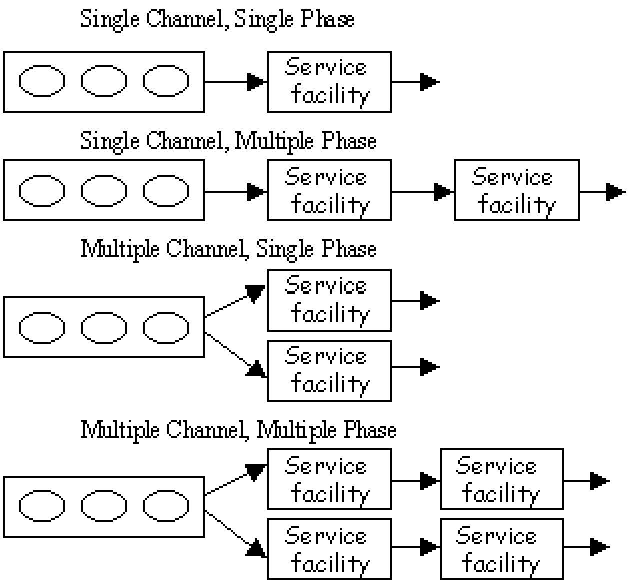
2. Queue: It is characterized by the maximum permissible number of units that it can contain. Queues may be infinite or finite.

3. Service Discipline: It refers to the order in which members of the queue are selected for service. Frequently, the discipline is first come, first served.

Following are some other disciplines:

- LIFO (Last In First Out)
- SIRO (Service In Random Order)
- Priority System

4. Service Mechanism: A specification of the service mechanism includes a description of time to complete a service and the number of customers who are satisfied at each service event. The service mechanism also prescribes the number and configuration of servers. If there is more than one service facility, the calling unit may receive service from a sequence of these. At a given facility, the unit enters one of the parallel service channels and is completely serviced by that server. Most elementary models assume one service facility with either one or a finite number of servers.The following figure shows the physical layout of service facilities.



Q8. Define Queuing theory and also discuss its assumptions and limitations.

Answer: Queuing Theory is the mathematical study of waiting lines or queues that enables mathematical analysis of several related processes, including arriving at the back of the queue, waiting in the queue and being served by the Service channels at the front of the queue.

Assumptions of Queuing Theory

- The source population has infinite size.
- The inter-arrival time has an exponential probability distribution with a mean arrival rate of l customer arrivals per unit time.
- There is no unusual customer behaviour.
- The service discipline is FIFO.
- The service time has an exponential probability distribution with a mean service rate of m service completions per unit time.
- The mean arrival rate is less than the mean service rate, i.e., l < m.
- There is no unusual server behaviour.

Limitations Of Queuing Theory

- The waiting space for the customer is usually limited.
- The arrival rate may be state dependent.
- The arrival process may not be stationary.
- The population of customers may not be infinite and the queuing discipline may not be First Come First Serve.

- Services may not be rendered continuously.
- The Queuing System may not have reached the steady state. It may be, instead, in transient state.

Q9. Explain Unusual Customer/Server Behaviour.

Answer: *Customer's Behaviour*

Balking. A customer may not like to join the queue due to long waiting line.

Reneging. A customer may leave the queue after waiting for sometime due to **impatience**.

Collusion. Several customers may cooperate and only one of them may stand in the queue.

Jockeying. When there are a number of queues, a customer may move from one queue to another in hope of receiving the service quickly.

Server's Behaviour

Failure. The service may be interrupted due to failure of a server (machinery).

Changing service rates. A server may speed up or slow down, depending on the number of customers in the queue. For example, when the queue is long, a server may speed up in response to the pressure. On the contrary, it may slow down if the queue is very small.

Batch processing. A server may service several customers simultaneously, a phenomenon known as batch processing.

Q10. A factory has 1000 bulbs installed. Cost of individual replacement is Rs. 3/- while that of group replacement Re. 1/-per bulb respectively. It is decided to replace all the bulbs simultaneously at fixed interval & also to replace the individual bulbs that fail in between. Determine optimal replacement policy. Failure probabilities are as given below:

Week	1	2	3	4	5
Failure Probability (P)	0.10	0.25	0.50	0.70	1.00

Solution:
The probabilities given in the problem are cumulative i.e. till week 1, till week 2 etc. Individual probabilities would be 0.10 in 1^{st} week, 0.15 (0.25-0.10) in 2^{nd} week, and so on. (as shown in the below table)

<u>Policy-I: Individual Replacement</u>
Step 1) Cost of Individual Replacements

Table 1

Week	Probability	Life (=time x probability)
1	0.10	1 x 0.10 = 0.10
2	0.15	2 x 0.15 = 0.30
3	0.25	3 x 0.25 = 0.75
4	0.20	4 x 0.20 = 0.80
5	0.30	5 x 0.30 = 1.50
Mean Life		3.45

Individual Failures/week = Total Quantity / Mean Life = 1000 / 3.45 = 289.9
Individual Replacement Cost = (Individual Failures per week) x (Individual replacement cost)
= 289.9 x 3 = **Rs. 869.6**

Policy-II: Group Replacement

Step 2) Individual failures per week
In the first week: 10 % (0.10) of the bulbs will fail out of 1000 bulbs i.e. 100
In the second week: 15 % of the bulbs will fail out of 1000 bulbs i.e. 150. Also, 10% of 100 replaced in the first week i.e. 10. TOTAL bulbs failed until second week = 160 (150+10)
Rest of the calculation is as shown in the below table:

Table 2

Wk	I*	II	III	IV	V	TOTAL
1	100.00					100.00
2	150.00	10.00				160.00
3	250.00	15.00	16.00			281.00
4	200.00	25.00	24.00	28.10		277.10
5	300.00	20.00	40.00	42.15	27.71	429.86
6	42.99**	41.57	70.25	32.00	30.00	216.80
7	21.68	64.48	69.28	56.20	48.00	259.63

* 10% of 1000, 15 % of 1000 etc.
** 429.86 x 0.10 = 42.99

Step 3) Calculating the total cost & time of replacement:

Table 3

Wk	Cumulative Failures*	Individual Failure Cost (Rs 3/bulb)**	Group Failure Cost	Total Cost (TC)	Average Cost (TC/Wk)
1	100.00	300.0	1000	1300.0	1300.0
2	260.00	780.0	1000	1780.0	890.0
3	541.00	1623.0	1000	2623.0	874.3
4	818.10	2454.3	1000	3454.3	**863.6**
5	1247.96	3743.9	1000	4743.9	948.8
6	1464.76	4394.3	1000	5394.3	899.0
7	1724.40	5173.2	1000	6173.2	881.9

* Adding the values of the last column of previous table
** Values of previous column by Rs 3.

Thus, replacing all the bulbs simultaneously at fixed interval & also to replace the individual bulbs that fail in between will be economical or optimal after 4 weeks (optimal interval between group replacements).

Interpretation:

1. The cost of only individual replacements id Rs. 869.6 (As seen in the Policy-I)
2. The cost of combine policy i.e. group and individual replacement id Rs. 863.6 (see last column of table 3)

3. Hence the Policy-II is the optimum replacement policy
Hence, the bulbs shall be replaced every four weeks individually as well as in groups which combine would cost Rs. 863.6 per week (lesser than individual cost of Rs. 869.6 per week)

Q11. Explain Dominance principle in game theory.
Answer: At times, a convex combination of two or more courses of action may dominate another course of action. Whenever a course of action (say A s or B q) is dominated by others, then that course of action (A s or B q) can be deleted from the pay-off matrix. Such a deletion will not affect the choice of the solution, but it reduces the order of the pay-off matrix. Successive reduction of the order using dominance property helps in solving games. In a game, sometimes a strategy available to a player might be found to be preferable to some other strategy / strategies. Such a strategy is said to dominate the other one(s). The rules of dominance are used to reduce the size of the payoff matrix. These rules help in deleting certain rows and/or columns of the payoff matrix, which are of lower priority to at least one of the remaining rows, and/or columns in terms of payoffs to both the players. Rows / columns once deleted will never be used for determining the optimal strategy for both the players.
This concept of domination is very usefully employed in simplifying the two – person zero sum games without saddle point. In general the following rules are used to reduce the size of payoff matrix.

PRINCIPLES OF DOMINANCE
Rule 1: If all the elements in a row (say ith row) of a payoff matrix are less than or equal to the corresponding elements of the other row (say jth row) then the player A will never choose the ith strategy then we say ith strategy is dominated by jth strategy and will delete the ith row.
Rule 2: If all the elements in a column (say rth column) of a payoff matrix are greater than or equal to the corresponding elements of the other column (say sth column) then the player B will never choose the rth strategy or in the other words the rth strategy is dominated by the sth strategy and we delete rth column .
Rule 3: A pure strategy may be dominated if it is inferior to average of two or more other pure strategies.

Q12. State the different types of Replacement Models.

Answer: In fact, in any system the efficacy (efficiency) of an item deteriorates with time. In such cases, either the old item should be replaced by a new item, or some kind of restorative action (maintenance) is necessary to restore the efficiency of the whole system.

The cost of maintenance depends upon a number of factors, and a stage comes at which the maintenance cost is so large that it is more profitable to replace the old item. Thus, there is a need to formulate the most effective replacement policy.

Replacement models are concerned with the problem of replacement of machines, individuals, capital assets, etc. due to their deteriorating efficiency, failure, or breakdown.

It is evident that the study of replacement is a field of application rather than a method of analysis. Actually, it is concerned with methods of comparing **alternative** replacement policies.

The various types of replacement problems can be broadly classified in following categories:

- Replacement of items whose efficiency deteriorates with time, e.g., machine, tools, etc.
- Replacement of items that fail suddenly and completely like electric bulbs & tubes.
- Replacement of human beings in an organisation or staffing problem.
- Replacement of items may be necessary due to new researches and methods; otherwise, the system may become outdated.

Q13. What is a game? Explain its characteristics.

Answer: A competitive situation is called a game.

Its characteristics are:

- Finite number of competitors: There are finite no. of competitors called players. The players need not be individuals, they can be groups, corporations, political parties, institutions or even nations.
- Finite number of action: a list of finite no. of possible courses of action is available to each player. The list need not be the same for each player.
- Knowledge of Alternatives: Each player has the knowledge of alternatives available to his opponent.
- Choice: Each player makes a choice, i.e., the game is played. The choices are assumed to be made simultaneously, so that no player knows his opponents' choice until he has decided his own course of action.
- Outcome or gain: The play is associated with an outcome known as gain. Here the loss is considered negative gain.
- Choice of Opponent: The possible gain or loss of each player depends upon not only the choice made by him but also the choice made by his opponent.

Q14. Define pure strategy and mixed strategy in the game theory.

Answer: Pure Strategy:

The simplest type of game is one where the best strategies for both players are **pure strategies**. This is the case if and only if, the pay-off matrix contains a saddle point. In a pure strategy, players adopt a strategy that provides the best payoffs. In other words, a pure strategy is the one that provides maximum profit or the best outcome to players. Therefore, it is regarded as the best strategy for every player of the game.

This is because if both of them increase the prices of their products, they would earn maximum profits. However, if only one of the organization increases the prices of its products, then it would incur losses. In such a case, an increase in prices is regarded as a pure strategy for the organization.

Mixed Strategy:

Mixed strategy means a situation where a saddle point does not exist, the maximin (minimax) principle for solving a game problem breaks down. In a mixed strategy, players adopt different strategies to get the possible outcome. For example, in cricket a bowler cannot throw the same type of ball every time because it makes the batsman aware about the type of ball. In such a case, the batsman may make more runs.

However, if the bowler throws the ball differently every time, then it may make the batsman puzzled about the type of ball, he would be getting the next time.

Q15. Define Maximum and Minimum Strategy.

Answer: Maximin Strategy:

As we know, the main aim of every organization is to earn maximum profit. However, in the highly competitive market, such as oligopoly, organizations strive to reduce the risk factor. This is done by adopting the strategy that increases the probability of minimum outcome. Such a strategy is termed as maximin strategy.

In other words, maximin strategy is the one in which a player or organization maximizes the probability of minimum profit so that the degree of risk can be reduced. Let us understand the maximin strategy with the help of an example. Suppose two organizations, A and B, want to launch a new product in a duopoly market.

The outcomes for these two organizations are shown in Table

Table-5: Payoff Matrix for Maximin Strategy

		Organization B		
Organization A		**No New Product**	**New Product**	**Organization A Minimum**
	No New Product	7,7	6,9	6
	New Product	9,6	4,4	4
	Organization B Minimum	6	4	

In Table, it is assumed that the main motive of both the organizations is to maximize their profits. Let us first analyze the outcome of organization B. Organization B would earn profit of Rs. 4 crores when both the organizations, A and B, launch a new product However, if only organization A launches a new product, then the profit of organization B would be Rs. 6 crores.

However, if organization B launches a new product, then it would earn profit of Rs. 4 crores. Therefore, the minimum gain of organization B is Rs. 4 crores after launching a new product. Similarly, the minimum gain of A is Rs. 4 crores by launching a new product. Maximin strategy is not used only for profit maximization problems, but it is also used for restricting the unrealistic and highly unfavorable outcomes.

For applying the maximin strategy, firstly, an organization needs to identify the minimum output or profit that it would get from a particular strategy. Table shows that the minimum output for organization A is Rs. 6 crores when it does not launch a new product. However, if it launches a new product, the minimum output would be Rs. 4 crores.

On the other hand, organization B also has the same amount of profit in both the cases. Now, both the organizations, A and B, would find out the strategy that would yield them maximum of the minimum output. In the present case, for both the organizations, A and B, it would be better if they do not launch any new product to yield maximum profit.

Minimax Strategy:

Minimax strategy is the one in which the main objective of a player is to minimize the loss and maximize the profit. It is a type of mixed strategy. Therefore, a player can adopt multiple strategies. It can be applied to complex as well as simple decision-making process. Let us understand the minimax strategy with the help of an example.

Suppose Mr. Ram wants to manufacture cream biscuits. For this, he selected three flavors, namely strawberry, chocolate, and pineapple, which he denoted with A, B, and C respectively He wants to select one of the flavors to produce cream biscuits and introduce them in the market on the basis of their demand.

He needs to predict the future events that can occur from the options he has selected. These future events are termed as the states of nature in decision analysis. The states of nature selected by Ram with respect to demand are high demand, medium demand, and low demand.

The payoff matrix for biscuits is shown in Table 6

Table-6: Payoff Matrix for Biscuits

Alternative Strategies	States of Nature		
	High Demand	Moderate Demand	Low Demand
A	400000	300000	-100000
B	550000	270000	-300000
C	300000	180000	-250000

Here, we are assuming that Mr. Ram adopts minimax strategy. Now, if he selects strategy A in a high demand market, then he would incur a loss of Rs. 150000. This is because he has not selected the strategy B that would yield maximum payoff of Rs. 550000.

In such a case, he would determine the maximum loss for each alternative and then select the alternative that would give minimum loss. Among each state of nature, the highest payoff is selected and subtracted from all other values in the state of nature.

Table-7 shows the loss or regret values of A, B, and C strategies:

Table-7: Regret Values

Alternative Strategies	States of Nature		
	High Demand	Moderate Demand	Low Demand
A	150000	0	0
B	0	30000	00000
C	250000	120000	150000

In Table-7, the maximum regret in each state of nature is highlighted with blue color. Among the highlighted regret values, strategy C has the least regret value of Rs. 120000. Therefore, Ram would select the strategy- C or pineapple flavor to produce biscuits.

Q16. What is a saddle point?

Answer: For a general two-player zero-sum game,

$$\max_{i \le m} \min_{j \le n} a_{ij} \le \min_{j \le n} \max_{i \le m} a_{ij}.$$

If the two are equal, then write

$$\max_{i \le m} \min_{j \le n} a_{ij} = \min_{j \le n} \max_{i \le m} a_{ij} \equiv v,$$

where v is called the value of the game. In this case, there exist optimal strategies for the first and second players.

A necessary and sufficient condition for a saddle point to exist is the presence of a payoff matrix element which is both a minimum of its row and a maximum of its column.
A game may have more than one saddle point, but all must have the same value.

Q-17Two companies, A and B, sell two brands of flu medicine. Company A advertises in radio (Ad,
Television (A2), and newspapers (A3). Company B, in addition to using radio (Bl), television

(B2), and newspapers (B3), also mails brochures (B4). Depending on the effectiveness of each
Advertising campaign, one company can capture a portion of the market from the other. Now summarize the percentage of the market captured or lost by company A.?

Sol.- B1 B2 B3 B4 Row min
.
A1 8 -2 9 -3 -3
A26 5 6 8 5Maximin
A3-2 4 -9 5 -9
Column max 8 5 9 8

Minimax

Q-18 What is game theory and properties of a game theory?

Ans-2 Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game,rather than just by the possibilities of several outcome results. Game theory does not insist onhow a game should be played but tells the procedure and principles by which action should be selected. Thus it is a decision theory useful in competitive situations. Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the game.
Going through the set of rules once by the participants defines a play.

Properties of a Game-:
1. There are finite numbers of competitors called 'players'
2. Each player has a finite number of possible courses of action called 'strategies'
3. All the strategies and their effects are known to the players but player does not know Which strategy is to be chosen.
4. A game is played when each player chooses one of his strategies. The strategies are Assumed to be made simultaneously with an outcome such that no player knows his Opponent's strategy until he decides his own strategy.
5. The game is a combination of the strategies and in certain units which determines the Gain or Loss.
6. The figures shown as the outcomes of strategies in a matrix form are called 'pay-off Matrix'.
7. The player playing the game always tries to choose the best course of action which Results in optimal pay off called 'optimal strategy'.
8. The expected pay off when all the players of the game follow their optimal strategies is Known as 'value of the game'. The main objective of a problem of a game is to find the Value of the game.

9. The game is said to be ‘fair’.

Q-19Solve the following pay-off matrix-

Player B

Player A		I	II	III	IV	V
	I	-2	0	0	5	3
Player A	II	3	2	1	2	2
	III	-4	-3	0	-2	6
	IV	5	3	-4	2	-6

Solution

Player B

Player A		I	II	III	IV	V	
	I	-2	0	0	5	3	-2
Player A	II	3	2	1	2	2	1 Maximin value
	III	-4	-3	0	-2	6	-4
	IV	5	3	-4	2	-6	-6
		5	3	1	5	6	

Minimax value

Strategy of player A – II
Strategy of player B - III
Value of the game = 1

Q-20 Classification of Game theory ?

All games are classified into-

- Pure strategy games
- Mixed strategy games

The method for solving these two types varies. By solving a game, we need to find best Strategies for both the players and also to find the value of the game.
Pure strategy games can be solved by saddle point method.
The different methods for solving a mixed strategy game are-

- Analytical method
- Graphical method
- Dominance rule
- Simplex method

Q-21 Two players A and B without showing each other, put on a table a coin with head or tail up.
Players A wins Rs.8 when both coins show head and Rs.1 when both coins are tails. B wins Rs.3 when the coins do not match. Given the choice of being matching player (A) or non-matching player (B), which one would you choose and what would be your strategy?

Solution: Since no saddle point is found, the optimal strategies will be the mixed strategies. The payoff matrix for A is found to be:

		Player B H	Player B T		
Player A	H	8	−3	4	$\frac{4}{11+4}=\frac{4}{15}$
	T	−3	1	11	$\frac{11}{11+4}=\frac{11}{15}$

Step 1: Taking the difference of two numbers in column I, we find 8 − (−3) = 11, and put it under column II.

Step 2: Taking the difference of two numbers in column II, we find (−3 −1) = − 4, and put the number 4 (neglecting the − ve sign) under column I.

Step 3: Repeat the above two steps for the two rows also.

Thus, for optimum gains, player A must use strategy H with probability 4/15 and strategy T with probability 11/15, while player B must use strategy H with probability 4/15 and strategy T with probability 11/15.

Step 4: To obtain the value of the game any of the following expressions may be used:

Using B's Oddments:

B plays H, value of the game,

$$v = ₹\ \frac{4\times 8+11\times(-3)}{11+4.} = ₹\left(-\frac{1}{15}\right)$$

B plays T, value of the game,

$$v = ₹\ \frac{4\times(-3)+11\times 1}{11+4} = ₹\left(-\frac{1}{15}\right)$$

Using A's Oddments:

A plays H, value of the game, $v = ₹\ \frac{4\times 8+11\times(-3)}{4+11} = ₹\left(-\frac{1}{15}\right)$

A plays T, value of the game, $v = ₹\ \frac{4\times(-3)+11\times 1}{4+11} = ₹\left(-\frac{1}{15}\right)$

The above values of v are equal only if the sum of the oddments vertically and horizontally are equal.

Thus the complete solution of the game is:

1) Optimum strategy for A is (4/15, 11/15), and for B is (4/15, 11/15).
2) Value of the game to A is v = ₹ (− 1/15) and to B is 1/15.

Q-22Two major parts P1 & P2 for a product require processing through six machine centres. The technological sequence of these parts on the six machines and the manufacturing times on each machine are:

Sol.-

P_1 Machine Sequence	**C**	**A**	**E**	**F**	**D**	**B**
Time (Hours)	2	3	4	5	6	1
P_2 Machine Sequence	B	A	E	F	C	D
Time (Hours)	3	2	5	3	2	3

What would be the optimal scheduling to minimize the total processing time for these two parts?

Solution: Constructing a two dimensional graph where the horizontal axis represents P_1 and the vertical axis represents P_2 one can shade the areas where both parts use the same machine as shown in the **figure 7.3.**

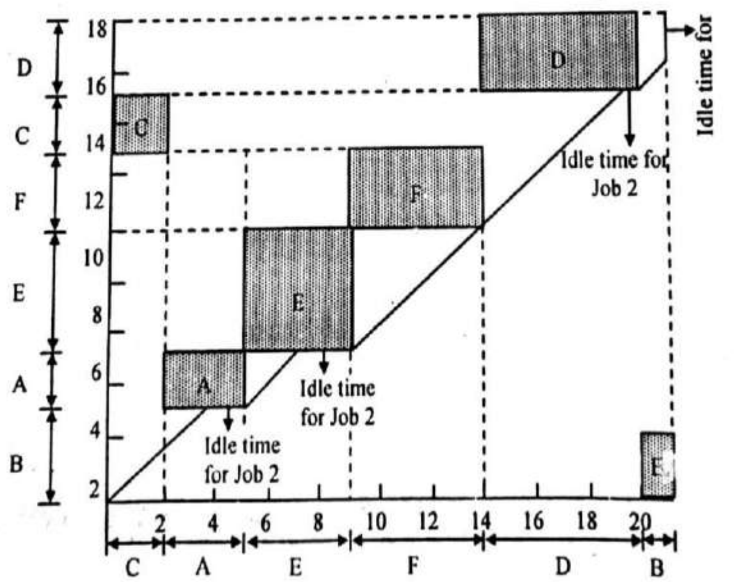


Figure 7.3: Graphical Representation

Lines 1 and 2 maximize the diagonal travel from the bottom left corner to the upper right corner

The total processing time is 23 hours.

Job I = 21 + idle time 2 = 23

Job II = 18 + idle time (2 + 2 + 1) = 23

Total Elapsed Time = 23.

<u>Q-</u>23 What do you mean by Sequencing. Explain advantages of sequencing?

Sol.- The process of determining the job order on some machine or in some work centre is known as sequencing. In other words, Job sequencing is the arrangement of the tasks required to be carried out sequentially.

The selection of an appropriate order for finite number of different jobs to be done on a finite number of machines is called sequencing problem.

Advantages of sequencing-:

- Minimizes the production cost.
- Less investment of material in process.
- Minimum material storage cost.
- Customer satisfaction.
- No over loading of men and machines.
- Good control of production.
- Improves goodwill of the company.
- Job satisfaction for the employees.

Q-24 Consider the following 2*2 game:

(4 7)
(6 5)

a) Does it have a saddle point?
b) Is it correct to state that the value of game, v will be $5<v<6$?
c) Determine the frequency of optimum strategies by matrix oddment method and find the value of the game?

Solution:

1) From the given pay-off matrix, we have maximin value = 5 and minimax value = 6.
Since maximin value ≠ minimax value, there does not exist a saddle point.

2) Since maximin value ≤ v ≤ minimax value, we must have 5 ≤ v ≤ 6.

3) Subtracting the second column from first, we get

$$C=\begin{bmatrix}-3\\1\end{bmatrix}$$

This gives $|C_1| = 1$ and $|C_2| = -3$

Now, subtracting the second row from first, we get

$$R = (-2 \;\; 2).$$

Thus $|R_1| = 2$ and $|R_2| = -2$

∴ The augmented pay-off matrix is:

			|Row Oddment|
	4	7	1
	6	5	3
|Column Oddment|	2	2	4

Since the sums of row oddments and column oddments each total 4, the optimum strategies of the two player are:

Row player: $\left(\frac{1}{4},\frac{3}{4}\right)$ and **Column player:** $\left(\frac{1}{2},\frac{1}{2}\right)$

The value of the game (v) $=\frac{1\times4+3\times6}{1+3}=\frac{4+18}{4}=\frac{11}{2}$

Q-25 Explain the Maximin and Minimax principle?

Sol.- 1) Maximin criteria- The maximizing player list is minimum gains from each strategy and selects the strategy, which gives the maximum out of these minimum gains.

2) Minimax criteria – The minimizing player list is maximum loss from each strategy and selects the strategy, which gives him minimum loss out of the maximum losses.

Maxi(min) = Mini(max) Optimal strategy

Q-26 What are the solutions to a game ?

Sol.- There may be games with saddle point or without the saddle point, and accordingly we have strategy for a game –

- For games with saddle point : PURE STRATEGY
- For games without saddle point: MIXED STRATEGY

The steps to find the solution of the game are shown below-

- Start
- Write the pay-off matrix

- Is there a pure strategy solution, if yes then formulate solve it. If no, then make sure it is a 2*2 game.
- Use dominance rule to reduce the matrix, if required.
- Solve by using the analytical formula mixed strategy.
- Then again check the 2*2 game.
- Use graphical method to reduce the problem into a 2*2 matrix.

Q27. Solve the following pay-off matrix, determine the optimal strategy and the value of the game:-

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